BUSINESS CYCLE FORECASTING

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Statistical study of the business cycle has long history. The two most important aspects of the analysis historically were related to its measuring and forecasting. In the literature (see BEA 1990, Lahiri and Moore 1990) is argued that the main measuring tools of the business cycle are the composite indicators – leading, coinciding and lagging. They are computed usually monthly and their components are specially selected. It is shown (see Lahiri and Wang, 1994) that the composite indicators in fact lead the business cycle and its turning points for the two main phases: recession and expansion. In regard to forecasting in this paper it is assumed that it could be done by forecasting the composite leading indicator.

The main goal of this paper is to define and apply a procedure for forecasting the composite leading business cycle indicator. This procedure combines some well known elements with relatively new ones like ranked forecasts, combined forecasts, etc.

Methodology

Every forecast contains certain error, which is measured by the difference between the forecast and the real value of the variable. Usually this error is measured as simple difference. Traditional methods of looking for one and only one "optimal" or "the best" forecast is considered inefficient and dangerous. It does not use all available information from all forecast. Not all possible forecast should be taken into account, but only those that brink new information not included in the other forecasts. The idea here is to use multiple forecasts approach and somehow to pool the information into a combined forecast that is manageable and easy to use.

In most cases there are several different forecasts that have to be compared using certain criteria, like mean error, mean square error[i], etc. The classical approach requires using one criterion to find the best model with the smallest error.

It is very difficult and very subjective in practice to choose a loss function in order to compare different models and select the best model. The danger is even bigger since in most cases different loss functions lead to different best models.

Regardless of our ability to find "the best" forecast with the smallest loss function there is a value in trying to use the information from more than one forecasts by combining them in one composite forecast. This process is like defining a portfolio of different types of financial assets in order to maintain a good financial health.

An important tool for combining forecasts is so called "encompassing" test. It offers the ability to determine whether one forecast incorporates (or encompasses) all the relevant information in the other forecasts we are trying to combine. The most popular encompassing test is done by running a regression with dependant variable the actual realization of the variable in question and all forecasts as right-hand side variables (see Chong and Hendry 1986) . For simplicity let's consider testing two forecasts: A and B. The regression model for the encompassing test is as follows:

$$Y_{t+k} = b_0 + b_1 \hat{Y}^{A}_{t+k,t} + b_2 \hat{Y}^{B}_{t+k,t} + e_{t+k,t}$$

Model A encompasses model B if $b_0 = b_2 = 0$ and $b_1 = 1$. Model B encompasses model A if $b_0 = b_1 = 0$ and $b_2 = 1$. In practice this almost never happens and for all other values of the coefficients neither of the models encompasses the other. If one of the models encompasses the other, of course, there is no need to combine the forecasts and we can just use the better model. Otherwise the combined forecast is preferable.

When the encompassing test fails to establish a better model the next logical step is to combine the forecasts so we can fully incorporate all the available information. Forecast combination techniques perform very well and in many cases are better than using only one forecast (see Clemen 1989).

The regression type of forecast combination is mostly based on the encompassing regression models, i.e. by regressing the realizations on forecasts. This method is very simple and flexible and depending on the type of regression used there are different types of methods for combining, like dynamic combining regressions etc. The calculation of the combined forecast is done by introducing the estimated values for all included forecast and using the encompassing regression to find the estimated values.

Forecasting Models

Data used in this study are the following: U.S. composite leading indicator (CLI), denoted with C_t and U.S. purchasing managers index (PMI) denoted with D_t. CLI is the official business cycle indicator while PMI is based on a survey similar to the "Current Economic Outlook" survey, administered by the Bulgarian National Statistical Institute [NSI, 1999]. Unfortunately the data for Bulgaria are relatively short and could not be used in this study. However, after accumulation of enough data point, the methods used here could be applied directly to the Bulgarian data.

The US data were tested and found to be integrated of order one and cointegrated [Vesselinov, 2001]. The seasonally adjusted data then were transformed by taking first differences of the log data.

The main goal of this section is to build five new models using the above described methodology and then generate forecasts with different horizons. In order to test the models quality the sample period will be divided into two: learning and testing. The learning period is January 1959 – December 1995, or 444 months all-together. These data are used to build the econometric models. The testing period is January, 1996 – June 1998, or 30 months in all. These data are used to compare the forecasts and real values. This way models' quality is evaluated by out-of-sample information.

Models [ii]

Except for Model 1, for the rest of the models the first differences of the original data are used.

Forecasting Model 1. Error-Correction Model for CLI (C_t).

Error-Correction models (ECM) are type of Vector-Autoregression models. ECM use the presence of cointegration between two series and establish their long-term equilibrium relationship. These models have statistical mechanism that kicks in when the two series begin to separate and there is a danger of violating the long-term equilibrium. Then the cointegrating equation becomes different from zero and brings together the two series

Preliminary tests confirmed that CLI (C_t) and PMI (D_t) are cointegrated of order 1, i.e. they are CI(1) and therefore ECM was a reasonable thing to do. Information criteria determined that the optimal ECM model has lag of one month, a constant and a trend. The model is presented below (standard error in parentheses) and the first part of the model presents C_t as dependent variable. The expression in square brackets [] is the cointegrating equation.

$$\Delta C_t = \underset{(0.00316)}{0.01192}\left[C_{t-1} - \underset{(0.14107)}{0.45162}\, D_{t-1} - \underset{(0.000\,07)}{0.000747}\, T - 2.54\right] +$$

$$+\underset{(0.05053)}{0.380654}\, \Delta C_{t-1} + \underset{(0.00371)}{0.003276}\, \Delta D_{t-1} + 0.000427$$

AIC = -8.49 SIC = -8.45

The second part presents D_t as dependent variable.

$$\Delta D_t = \underset{(0.04083)}{0.240634}\left[C_{t-1} - \underset{(0.14107)}{0.45162}\, D_{t-1} - \underset{(0.00007)}{0.000747}\, T - 2.54\right] +$$

$$+\underset{(0.65274)}{6.775174}\, \Delta C_{t-1} + \underset{(0.04792)}{0.005385}\, \Delta D_{t-1} - 0.005575$$

AIC = -3.37 SIC = -3.33

All coefficients of the above equations are statistically significant.

Impulse response function for CLI shows that one standard deviation shock in PMI leads to 0.5% change during one-year period and similar shock in CLI has even smaller effect. The response function for PMI indicates that one standard deviation shock in CLI leads to 5% change for the first 3-4 months and then the influence decreases to 2%. One standard shock in its own series leads to 4% change and linearly declining afterwards toward zero.

Variance decomposition of CLI shows that the influence of PMI varies between 1% and 8%. The influence of CLI on PMI's variance is considerably larger, from 15-20% in the beginning of the period to 60% toward the end of the period.

Forecasting Model 2. EGARCH(1,1) Model of CLI (C_t).

The preliminary tests for presence of ARCH effects determined that such effect exists in the residuals of the model with probability of error in the range of 0.001 and therefore the GARCH methodology is applicable here.

The optimal GARCH model for CLI (C_t) is exponential GARCH or EGARCH(1,1). Conditional mean equation is as follows (standard error in parentheses):

$$C_t = 0.034339 + \underset{(0.048092)}{0.299126}\, C_{t-1} + \underset{(0.052377)}{0.152555}\, C_{t-2}$$

Conditional variance equation is presented below.

$$\log\left(s_t^2\right) = -0.135032 + \underset{(0.011419)}{0.969002}\, \log\left(s_{t-1}^2\right) + \underset{(0.041633)}{0.084487}\left|\frac{e_{t-1}}{s_{t-1}}\right| - \underset{(0.022185)}{0.114482}\left[\frac{e_{t-1}}{s_{t-1}}\right]$$

AIC = 0.601 SIC = 0.666

The test for normality of the residuals is JB = 58.2, $p < 0.001$, i.e. the residuals are normally distributed. The test for ARCH effect is negative: F-statistics = 0.1319, $p = 0.717$ and Engel's LM statistics = 0.1324, $p = 0.716$, i.e. by building the EGARCH model the heteroscedasticity is eliminated.

All coefficients are statistically significant. From the first equation is evident that the two lag values have positive effect on current value. The leverage effect is exponential and not quadratic and since the coefficient before the relative shock is negative (-0.114482) then the presence of the effect is confirmed. Statistical significance of this coefficient means that this effect is asymmetrical.

<u>Forecasting Model 3. TARCH-M(1,1) Model of CLI (C_t).</u>

A slightly different GARCH model is built here. The difference from Model 2 is that current and lag values of PMI are present in the right-hand side of the model. The optimal model here was the threshold ARCH, or TARCH-? (1,1) model. The conditional variance presented by the standard deviation is included in the conditional mean equation:

$$C_t = 0.466916 + \underset{(0.002574)}{0.029042}\, D_t + \underset{(0.002836)}{0.011077}\, D_{t-1} - \underset{(0.284135)}{1.301105}\, s_t$$

Conditional variance equation is as follows:

$$s_t^2 = 0.005762 - \underset{(0.016750)}{0.042704}\, e_{t-1}^2 + \underset{(0.042551)}{0.170983}\, e_{t-1}^2 d_{t-1} + \underset{(0.035839)}{0.899588}\, s_{t-1}^2$$

AIC = 0.466   SIC = 0.540

The test for normality is JB = 21.06, $p < 0.001$, i.e. residuals are normally distributed. The test for ARCH effects is negative: F-statistics = 0.545, $p = 0.461$ and Engel's LM statistics = 0.547, $p = 0.460$, which indicates that the heteroscedasticity is overcome.

Strangely enough, in the optimal model there are no lag values of the left-hand variable (they were non-significant) and only current and lag values of PMI are present. The leverage effect is $g = +0.170983$, i.e. it is positive and statistically significant. This means that the effect exists and different type of news have different effect on conditional variance. The good news ($e_t < 0$) have effect in the range of $a = -0.043$, while the bad news have effect of $a + g = 0.112$ on conditional variance. In other words, the good news make CLI more stable, less changeable, while the bad news have larger influence and lead to instability, which is understandable and confirmed in real life.

<u>Forecasting Model 4. State-space Model of CLI (C_t).</u>

The optimal model for C_t has one lag for the observed variable and one lag for the state unobservable variable. All coefficients are statistically significant.

Observation equation:

$$C_t = 0.031629 + \underset{(0.047477)}{0.533212}\, C_{t-1} + S_t$$

State equation:

$$S_t = \underset{(0.052394)}{-0.169306}\, S_{t-1}$$

The lag value of CLI has positive effect on the current value, and the lag value of the state variable has negative effect.

Forecasting Model 5. Second State-Space Model for CLI (C_t).

The difference from the previous model is that here PMI current and lag values are included in the model in addition to CLI series. The optimal model of this kind has one lag for the observed variable and 3 lags for the state variable. All coefficients are statistically significant.

Observation equation:

$$C_t = 0.111916 - \underset{(0.0626634)}{0.588873}\, C_{t-1} + \underset{(0.002294)}{0.0304}\, D_t + \underset{(0.003419)}{0.02172}\, D_{t-1} + S_t$$

State equation:

$$S_t = \underset{(0.06001)}{0.744068}\, S_{t-1} - \underset{(0.074488)}{0.392328}\, S_{t-2} + \underset{(0.054519)}{0.293394}\, S_{t-3}$$

The model indicates that when PMI current and lag values are present then the lag values of CLI have negative effect.

Using the above five models forecasts are made for 6, 12 and 30 months ahead and the forecasts are compared to the real values.

Results

The existence of 5 different forecasts (based on five models) leads to the necessity of applying the encompassing test. The model of the encompassing test of Chong and Hendry for the five forecasts is presented below (standard error in parentheses).

$$\hat{Y}_t = \underset{(0.003869)}{-0.153634} + \underset{(0.013398)}{0.079255}\, F_{1,t} + \underset{(0.014105)}{0.184747}\, F_{2,t} + \underset{(0.010699)}{0.445289}\, F_{3,t} - \underset{(0.038646)}{3.17179}\, F_{4,t} + \underset{(0.007008)}{0.404389}\, F_{5,t}$$

,

where $F_{1,t}$ – forecast of Model 1, $F_{2,t}$ – forecast of Model 2, $F_{3,t}$ – forecast of Model 3, $F_{4,t}$ – forecast of Model 4, $F_{5,t}$ – forecast of Model 5. All coefficients, including the constant are statistically significant at $p<0.001$. This means that none of the forecasts incorporates another forecast, or in other words there is no redundant forecast. All forecasts are sub-optimal and all of them are necessary in order to reflect all the useful and unique information. That is the reason to continue further with combining those 5 forecasts based on the encompassing regression model and the new forecast is called "combined forecast."

Place Table 1 here.

Table 1 presents the results and forecasting accuracy of the five models plus the combined forecast. Three forecasting horizons are used: short-term of 6 months, medium term of 12 months, and long-term forecasts of 30 months. The idea is to give the models the opportunity to be tested in diverse situations since some of them may be better in short-term and other – in long-term forecasting.

The best model for given criterion gets rank 1, and the worst – gets rank 6. These ranks are averaged for each of the horizons and for the whole period. For example, from the firs row of Table 1 it can be concluded that according to ME criterion the combined forecast is the best and it gets rank 1, while forecast # 3 is the worst and gets rank 6.

According to the mean ranks, in short-term the best forecast is # 5 and the combined forecast with mean ranks 2.5 and 2.6 respectively. The worst forecast is # 3 with mean rank of 5.2. For the medium-range the best forecast is the combined forecast with mean rank of 2.2, and the worst - # 3 with mean rank 6. For the long-term scenario the best forecast is the combined and the worst – again # 3. Finally, for the whole period of 30 months and 5 criteria the best forecast evidently is the combined forecast with total average rank of 2.3. Second best is forecast # 2 with mean rank of 3.0. The worst forecast is without doubt forecast # 3 with mean rank of 5.7. The combined forecast is the best in middle and long-range and shares the first place for the short-term forecasts.

The forecasts for 30 month period between January, 1996 and June 1998 are presented on Figures 1-6. Figure 3 for example shows why forecast #3 had the worst characteristics. The last Figure 6 is proof in favor of combined forecasts: although none of the five single forecasts was optimal and exceptional, their combined forecast is extremely accurate for the whole 30 month period.

In conclusion we have to note that the methodology used in this study works very well and the resulting combined forecast has very good characteristics. In another work [Vesselinov, 2001] we present successfully similar models for Bulgarian business cycle using the “Confidence in Industry Index” and “Business Climate Index” statistics published by Bulgarian National Statistical Institute. The only obstacle left is the insufficient length of the Bulgarian time series which prevent us of dividing the period in two sub-periods. When enough information is accumulated the methodology applied here could be used successfully to make forecasts for Bulgarian business cycle.

Table 1. Ranks of the Forecasts.

Forecast Horizon	Criterion	Forecast/Model 1	2	3	4	5	Combined
6 months	ME	3	4	6	5	2	1
	MPE	3	4	2	5	1	6
	MSE	5	4	6	2	3	1
	MSPE	2	1	6	5	3	4
	Theil	5	5	6	2	3	1
	Average	3.6	3.4	5.2	3.8	2.4	2.6
12 months	ME	2	3	6	4	5	1
	MPE	1	2	6	3	5	4
	MSE	5	4	6	2	3	1
	MSPE	2	1	6	3	5	4
	Theil	5	4	6	2	3	1
	Average	3	2.8	6	2.8	4.2	2.2
30 months	ME	4	2	6	5	3	1
	MPE	3	2	6	4	1	5
	MSE	5	4	6	2	3	1
	MSPE	1	2	6	4	5	3
	Theil	5	4	6	2	3	1
	Average	3.6	2.8	6	3.4	3	2.2
Total	Average	3.4	3	5.7	3.3	3.2	2.3
	Total Rank	5	2	6	4	3	1

Figure 1. Composite Leading Indicator (CLI).
Model 1 Forecast.

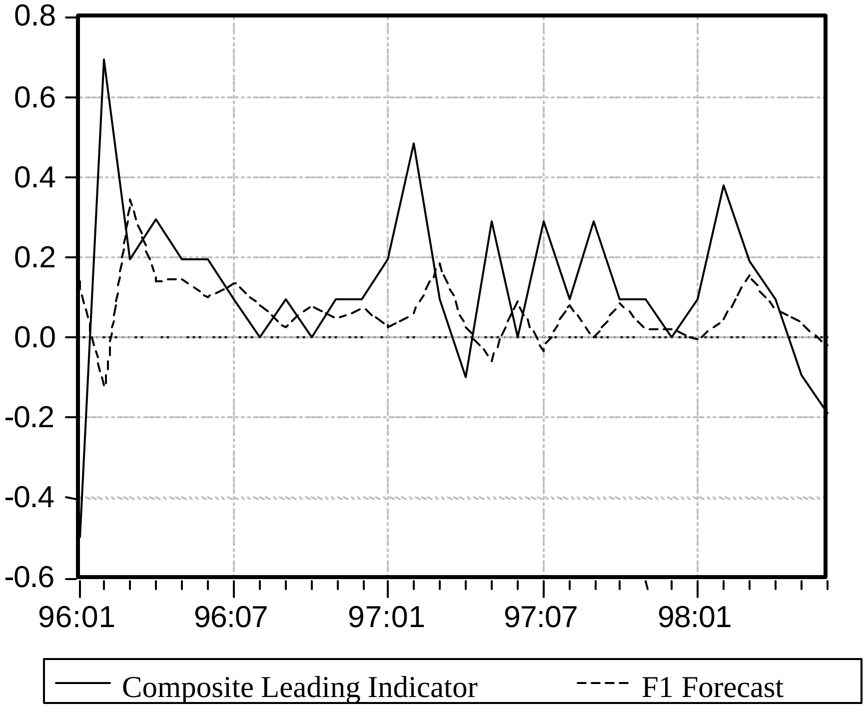


Figure 2. Composite Leading Indicator (CLI).
Model 2 Forecast.

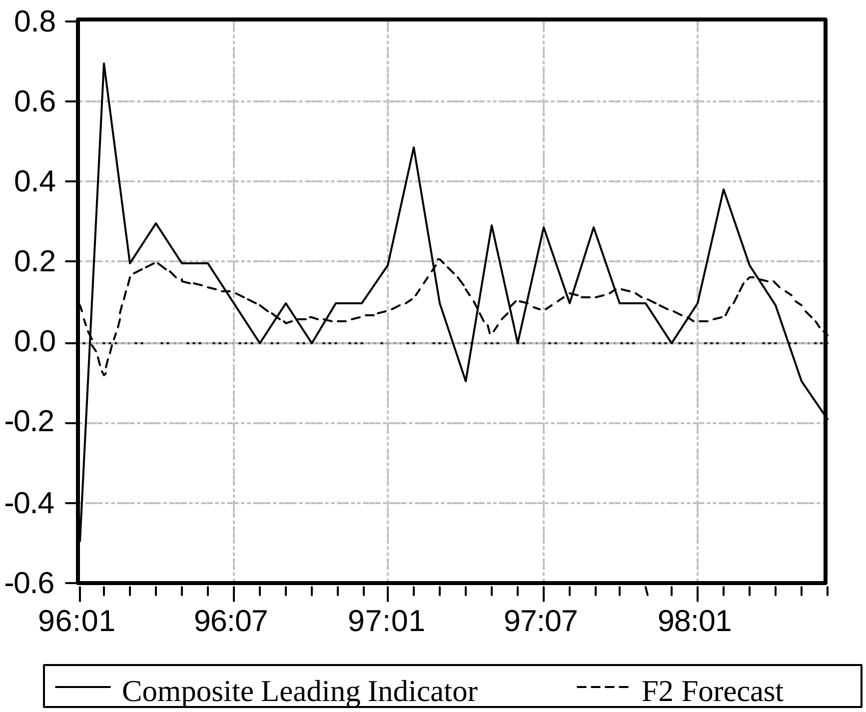


Figure 3. Composite Leading Indicator (CLI).
Model 3 Forecasts.

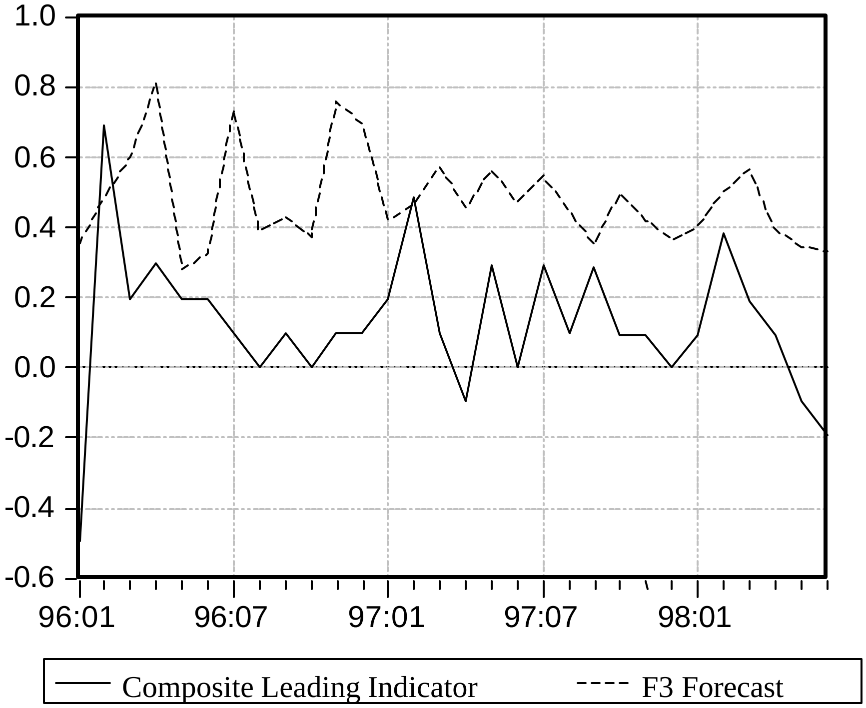


Figure 4. Composite Leading Indicator (CLI).
Model 4 Forecast.

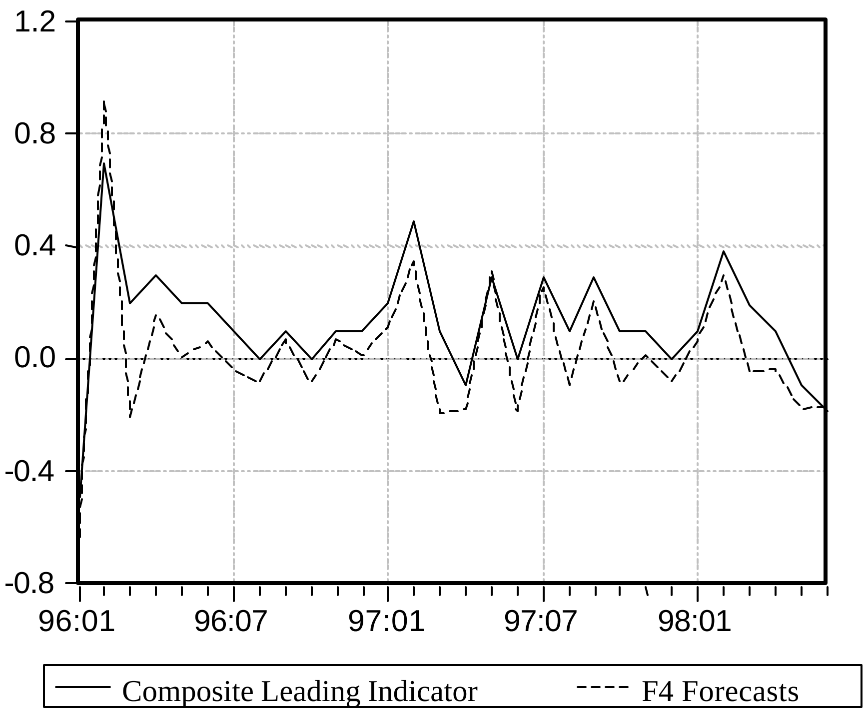


Figure 5. Composite Leading Indicator (CLI).
Model 5 Forecast.

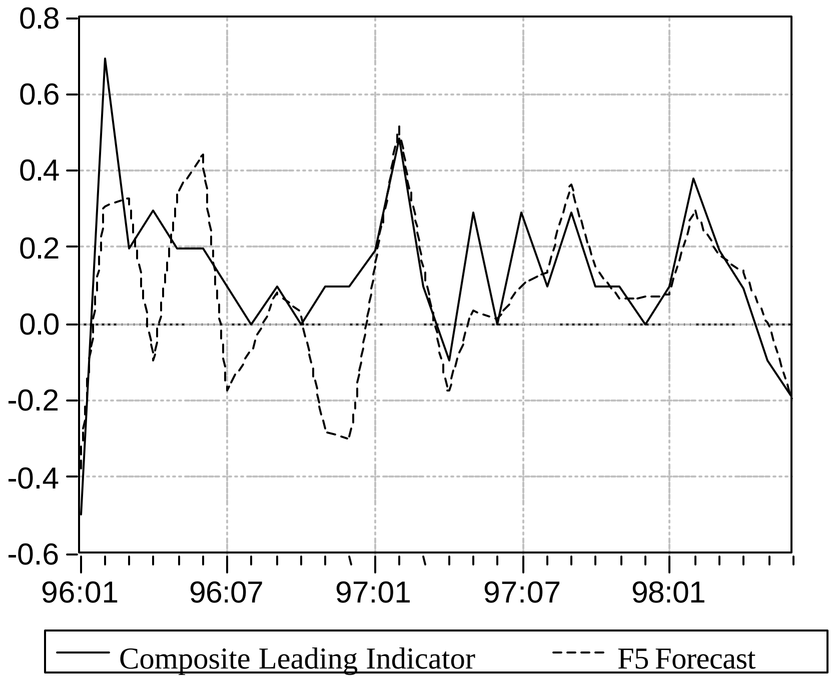
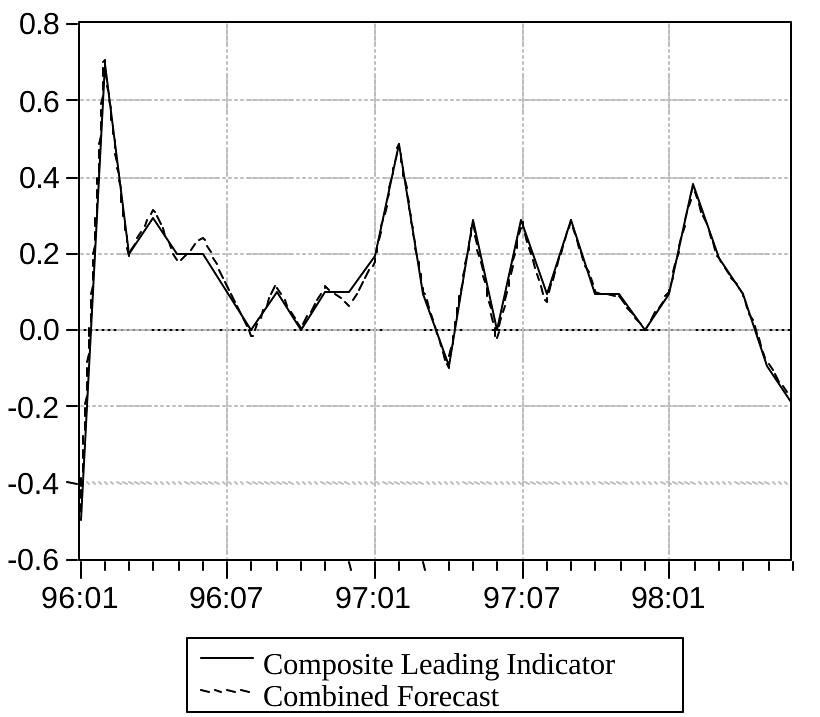


Figure 6. Composite Leading Indicator (CLI).
Combined Forecast.



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Notes:

[i] The following concepts are used in the paper. The forecasting error is defined as:

$e_{t+k,t} = Y_{t+k} - \hat{Y}_{t+k,t}$, where Y_{t+k} are real values for the period (t+k), and $\hat{Y}_{t+k,t}$ are the forecasts for the same period (t+k), made at time t.

The percent error is defined as follows:

$$p_{t+k,t} = \frac{Y_{t+k} - \hat{Y}_{t+k,t}}{Y_{t+k}}$$

Measures of bias as one of the component of the accuracy are given by the classic loss functions:

$$ME = \frac{1}{T}\sum_{t=1}^{T} e_{t+k,t}$$ - Mean Error

$$MPE = \frac{1}{T}\sum_{k=1}^{T} p_{t+k,t}$$ - Mean Percent Error

$$MSE = \frac{1}{T}\sum_{k=1}^{T} e_{t+k,t}^{2}$$ - Mean Squared Error

$$MSPE = \frac{1}{T}\sum_{k=1}^{T} p_{t+k,t}^{2}$$ - Mean Squared Percent Error

$$U = \frac{\sum_{k=1}^{T}\left(Y_{t+k} - \hat{Y}_{t+k,t}\right)^2}{\sum_{k=1}^{T}\left(Y_{t+k} - Y_t\right)^2}$$ - Theil's U statistics:

[ii] More details could be found in Vesselinov, 2001. Each and every one of the 5 models is result of a selection procedure. Series of models with different lags, or structure were estimated and compared using information criteria – Akaike (AIC) and/or Schwarz (SIC):

$$AIC(\text{p}) = n\log\left(\hat{s}^{2}\right) + 2\,\text{p}$$

$$SIC(\text{p}) = n\log\left(\hat{s}^{2}\right) + \text{p}\log\left(n\right),$$

where, n – number of observations, $\hat{s}^{2} = \frac{RSS}{n-p}$, RSS = $\Sigma\, \hat{e}_t^2$ - sum of squared residuals, and p – total number of parameters. Both criteria punish for including more parameters and the models with the smallest information criteria are considered "the best."